GOVERNMENT DEGREE COLLEGE

VEERAGHATTAM

STUDENT STUDY PROJECT

ON

BIOGRAPHY OF J.J. SYLIVESTER

DEPARTMENT OF MATHEMATICS

SUBMITTED TO

SRI.CH.NAIDU

LECTURER IN MATHEMATICS

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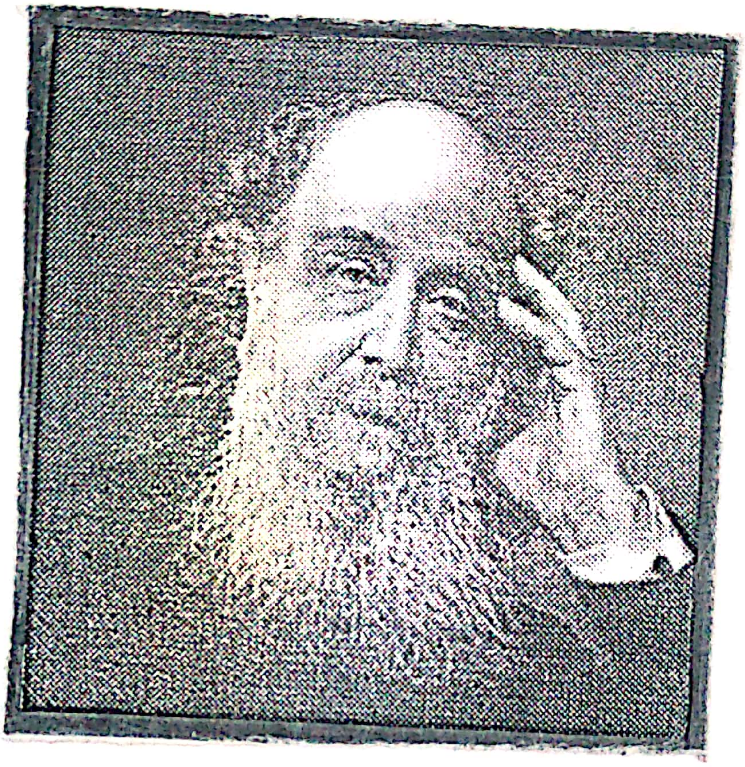
A MATHEMATICAL CENTENARY

J.J. Sylvester 1814-1897



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J. J. Sylvester was born James Joseph
on 3 september 1814, adopting the surname -
Sylvester later in life - following the lead of his
oldest brother. His mathematical skills were
evident early and by the age of fourteen he
was a student of Augustus de Morgan at uni-
-versity college London, which had only just opened
1 in 1828. From 1829 to 1831 he studied at the
Royal Institution at Liverpool, where the Examiner's
reports placed him in a special class of his own.
In 1831, at the age of 17 years, Sylvester went to
St John's college cambridge but, due to health -
problems, did not take the mathematical
tripos until 1837. At that time the can-
-didates were placed in order of merit by
the Examiner's. A practice that continued
untill early this century. Sylvester was
placed second in the list; the man who
came first never did any more mathematics
mathematics.

In 1838 JJS was appointed professor of natural philosophy (essentially, physics) at university college London, where his former teacher de-Morgan was a colleage. The following year, at the age of 25, he was elected to the Royal society At this point his future carreer apeared well set up. but his skills were mathematical rather than scientific and he found the teaching of scientific courses irksome. He resigned from his post at UCL and took up the post before of professor of Mathematics at the university of viriginia in 1841. He spent only three months in this post before resingning.

New man (P. 340 of reference 7) summeries the events. which took place in one of the southern states of America some 20 years before the American civil war, as follows.

"One day a young member of the chinarly whose classroom recitation he had citicized prepared an ambush and fell upon sylvester with a heavy wakink with a heavy walking stick. He (JJS) speared the student with a sword cane; the damage was slight, but the professor found it advisable to leave his post and take the earliest possible passage for England".

His American experiences had disillusioned him on the prospects teaching and he took up Employment as an actuary for a life insurance Company. In 1846 he set out upon a legal career, entering the Inner temple at the age of 32, and was called to the Bar in 1850. During these wilderness years. JJS had remained mathe-matically active by taking private pupils including rather surprisingly, Florence Nightingale

(3)

In 1852 JJS made the acquaintance of another mathematician who had stumbled into the wrong profession law. this was Arthur cayley. remembered for the cayley. Hamilton theorem, amongst other things. the two men had a common interest in developing the theory of algebraic invariants ~~which~~ which arose from the earlier work of Boole in 1841. (A simple example of an algebraic invariant is the discriminant $d = b^2 - 4ac$ of the quadratic polynomial $ax^2 + bx + c$).

Two years later, JJS applied, unsuccessfully for the professorship of mathematices to the Royal Military college. woolwich. He was appointed to the post the following year after the death of the succenful candidate, and remained there until 1870 when he was compelled to retire at the age of 56 in some what acrimained there circumstances. During this tenure he was elected to the French Academy of sciences in 1863. One his retirement he pursued his many intre-sts which included reading the classics. playing chess and versifying. In 1870 he published classics. playing chess and versifying In 1870 he publised a pamphet. the laws of verse.

The john Hoplins university was founded, at Baltimore, Maryland, in 1875. H.F. Baker (references) Comments.

"The authorites seem to have felt that a professor of Mathematics and a professor of classics could inaugurate the work of an (sic) universty without expensive buildings or elaborate apparatus (plus ca change... RJC). It was finally agreed that sylvester should go, securing, besides his travelling expenses, an annual stipend of 5000 dollars "paid in gold".

This stipend represented a considerable sum of that time and J J S remained at Johns Hopkins untill 1883, revitalized as a mathematician. The American Journed of Mathematics was founded by J J S in 1878 and it remains one of the worlds leading research journals. In its early years it did much to stimulate mathematical research in the united states.

3 The eminent number theorist H.J.S Smith died in 1883, at the age of 57. Oxford university invited sylvester, by then aged 70, to take the vacant post of savilian professor of Geometry. thus sylvester returned to england and delivered his inaugural lecture, at the age of 71, on 12 Dec 1885. In 1869, in a presidental address to the British Association, he had declared 'the Mathematicians lives long and live young'. He was living proof of this, continuting in post at oxford until 1894 when failing eyesight forced him into retirment som 24 years after he was compelled to retire from the post at woolwith. In march 1897 he suffered from a stroke and died a few days later on 15 march

Bell describes sylverster as short and stocky with a fiery presonality and broad intellectual interests. He read the classics and Frech, German and Italian literature in the original, illuminating many of his papers with apt qudations. His inte-rest in language led him to introduce much of the terminology in mathematics, in par-ticular he is credited with having introduced the term matrix into mathematical termi-nology.

6 His interests in Mathematics were also wide ranging. Much of the work he did on matrices is still relevant to current research in linear programming Nearly a century and a half later, one part of sylversters work, dating from 1852, still survives in many undergraduate mathematics courses. Despite its name, sylversters low of intertia is a theorem in linear algebra, or Gemetry, rather than mechanics mechanics.

When a quadratio from of rank r is reduced by two real non-singular linear transformations to the diagonal forms.

$$a_1 x_1^2 + \cdots + a_r x_r^2 \qquad b_1 x_1^2 + \cdots + b_r x_r^2$$

$a_1x_1^2 + \ldots\ldots + a_r x_r^2$ $\quad b_1x_1^2 + \ldots\ldots + b_r x_r^2$

the number of positive a's is the same as the number of positive b's.

In number theory a positive integer n is called perfect when the sum of its divisors is 2n. The first few perfect numbers are 6, 28, 496, 8128,.... and it is known that an even number is perfect if and only if it is of the form.

$$2^{p-1}(2^p - 1)$$

where $2^p - 1$ (and therefore also p) is prime. It is still not known whether there are any odd perfect numbers. In 1888 sylvester showed that any perfect number must have atleast five different prime divisors. All we know along these lines, over a century later and with the advantage of massive computing power, is that such numbers must have atleast eight different prime divisors.

Andrews draws attention to Sylver's work on partitions, a subject first considered systematically by euler. A partition

A Partition of a positive integer n is a expression of n as a sum of positive integers. For ex $7 = 4+3$ or $7 = 3+2+1+1$ and such partitions can be exhibited graphically by the Ferrers graph. For example,

X X X X

X X X

illustrates the paratition $7 = 4+3$. counting points vertically we also have the conjugate partition: $7 = 2+2+2+1$. A Partition is called self-conjugate partitions for example $6 = 3+2+1$ is self-conjugate.

It is not too difficult if you are Euler, to see that the number of partitions $P(n)$ of n can be generated by the formula.

$$\sum_{n=1}^{\infty} P(n)x^n = \prod_{n=1}^{\infty} (1 - x^n)^{-1}$$

Since $(1-x^n)^{-1} = 1 + x^n + x^{2n} + x^{3n} + \cdots$ (If you lack Euler's insight see ch. 14 of reference 2 or ch. XIX of reference 5.) In 1750 Euler considered the reciprocal of this generating function.

$$\prod_{n=1}^{\infty} (1 - x^n)$$

and obtained a series expansion for it which is called Euler's pentagonal number theorem. By considering certain self-conjugate

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self-conjugate partitions Sylvester was able to generalize eulers pentagonal number theorem. This led F. Franklin to develop a purely combinatorial proof of the pentagonal number theorem. arising so naturally from sylvesters work that it is referred to as the Franklin-Sylvester method. This proof is described as 'remarkable' by Apostol, 'very beautiful' by Hardy and wright and 'Probably the first major discovery in American mathematics' by Andrews. A more detailed biography is provided by Bell (reference 4), which I have used extensively in preparing this article.

REFERENCES

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③ E.T. Bell, Men of Mathematics vol. 2 (Penguin, London, 1953).

④ J.R Newman, The would of Mathematics vol. 1 (Simson and Schuster, New York, 1956).